BCA 1st Semester (Honours) Examination, 2021
BACHELOR OF COMPUTER APPLICATION

Course ID : 13314 **Course Code : BCA-GE-01**

Course Title : Mathematics-I

Time : 3 Hours **Full Marks : 80**

The figure in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group-A

1. **Answer all the questions from the following. Choose Correct Options:** **10 X 1 = 10**

i. The value of $\frac{(\cos 3\theta + i\ \sin 3\theta)^7 (\cos 5\theta - i\ \sin 5\theta)^4}{(\cos 4\theta + i\ \sin 4\theta)^{10} (\cos 13\theta - i\ \sin 13\theta)^3}$ **is**

a) 0 b) 1 c) -1 d) None of these

ii. The remainder when $4x^4 - 10x^2 + 1$ is divided by x-2 is

a) 15 b) 10 c) 25 d) None of these

iii. The solution of the equation $\begin{vmatrix} x-2 & 2 & 5 \\ x-7 & 3 & 6 \\ 2x-6 & 4 & 7 \end{vmatrix} = 0$ **is given by**

a) x=6 b) x=-6 c) x=0 d) None of these

iv. **The set of non-zero real numbers w.r.t. usual multiplication is**

a) not a group b) a non-abelian group c) an abelian group d) None of these

v. The transformed equation of $\frac{x}{3}+\frac{y}{4}=2$ **when the origin is transferred to the point (3,4) is**

a) $\frac{x}{4}+\frac{y}{3}=0$ b) $\frac{x}{3}+\frac{y}{4}=0$ c) $\frac{x}{3}-\frac{y}{4}=0$ d) None of these

vi. The centre of the conic given by $3x^2 - 8xy + 7y^2 - 4x + 2y - 7 = 0,$ **is**

a) (1,1) b) (1,2) c) (2,1) d) None of these

vii. The values of λ and μ for which the vectors
$3\hat{i} + 4\hat{j} + \lambda\hat{k}$ and $\mu\hat{i} + 8\hat{j} + 6\hat{k}$ **are collinear is given by**

a) **λ = 3 , μ=-6** b) **λ =3, μ=6** c) **λ =6, μ =3** d) None of these

viii. The set of real numbers R with respect to usual addition and multification is

a) not form a vector space over the set of reals R
b) not form a vector space over the set of rational Q
c) not form a vector space over the set of complex c
d) None of these

ix. The set of numbers of the form $a + b\sqrt{2}$, where a and b are rational number, w.r.t usual addition and multiplication is

a) a field b) not a Ring c) not a field d) None of these

x. if A = $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ **and B =** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$**, then**

a) A and B both singular b) A and B both non-singular c) A is singular and B is non-singular d) None of these.

Group-B

2. **Answer <u>any ten</u> questions from the following.** **10 X 2 = 20**

i. Prove that $\frac{z-i}{z-1}$ is purely imaginary then the point z lies on the circle whose centre is at the point $\frac{1}{2}(1+i)$and radius is $\frac{1}{\sqrt{2}}$

ii. Find the cube roots of unity.

iii. Divide $x^6 - x^4 + x^3 + 5x + 6$ by $x^2 - 2x + 3$, by synthetic devision.

iv. Without expending, show that $\begin{vmatrix} 2 & 0 & 1 \\ 1 & 3 & 0 \\ 0 & 1 & 4 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 2 \\ 3 & 0 & 1 \\ 1 & 4 & 0 \end{vmatrix}$

v. Determine the matrices A and B where A + 2B=$\begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and 2A –B = $\begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

vi. If A={1,2,3}, B={3,4,5,6} and c={1,2,7,5} find (A X B) ∩ (B X C)

vii. Let $f : R \rightarrow R(set\ of\ reals)$ be defined by $f(x) = x^2 + x - 2$ find $f^{-1}(10)$ and f(f(-2))

viii. Do the set of integers form a multiplicative group?

ix. Find the nature of the conic $\frac{8}{r} = 4 - 5\cos\theta$

x. Find the angle of rotation of the coordinate axes about the origin which will transform the equation $x^2 - y^2 = 4$ to $x^/ y^/ = 2$

xi. Show that the curve $4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$ has no centre.

xii. Find a unit vectors parallel to the resultant of two vectors $4\hat{\imath} + 2\hat{\jmath} - 5\hat{k}\ and\ 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

xiii. Are the vectors $\hat{\imath} + \hat{k},\ 2\hat{\jmath} + 4\hat{k}, and - \hat{\imath} + \hat{\jmath} + \hat{k}$ coplanar?

xiv. Find the equation whose roots are $2 \pm \sqrt{3}, 5 \pm \sqrt{6}$

xv. Find the conditions that the roots of the equation $x^3 - px^2 + qx - r = 0$ will be G.P.

Group-C

3. Answer <u>any four</u> questions from the following. **5 X 4 = 20**

i. Solve $8x^3 - 36x^2 + 42x - 5 = 0$ by Cardan's Method .

ii. Solve by Cramer's rule

$-x + y + z = 2$

$2x - y + 3z = 4$

$3x + 2y - 6z = 1$

iii. Let T be the linear transformation on R^3 to R^3 defined by T(x,y,z) = (2x, 4x-y, 2x+ 3y –z). Show that T is invertible and find T^{-1}

iv. Define 'Ring' and show that the set of all even integers form a commutative ring.

v. Reduce the equation $7x^2 - 6xy - y^2 + 4x - 4y - 2 = 0$ to the canonical form and find the nature of the conic.

vi. Prove by vector method, that the medians of a triangle are concurrent.

Group-D

4. **Answer <u>any three</u> questions from the following.** **10 X 3 = 30**

a. i. Show that the vectors $-5\vec{a} - 12\vec{b} + 4\vec{c}$, $4\vec{a} - 5\vec{b} + 8\vec{c}$ and $13\vec{a} + 2\vec{b} + 12\vec{c}$ are collinear.

ii. If the diagonals of a quadrilateral bisects each other, show, by vector method, the figure is parallelogram.

b. i. Find the point on the conic $\frac{l}{r} = 1 - \cos\theta$ which has the smallest radius vector.

ii. Find where the origin is to be shifted without changing the direction of the axes in order that the terms in x and may be removed from the equation $x^2 - y^2 - 8x - 6y + 7 = 0$

c. i. Solve the equation $x^4 - 12x^3 + 48x^2 - 72x + 35 = 0$ by removing the second term.

ii. Show that $x^7 + 5x^4 - 3x + k = 0$ has at least four imaginary roots.

d. i. Show that product of all values of $(1 + i\sqrt{3})^{3/4}$ is 8.

ii. Solve: $x^7 - 1 = 0$

e. i. Find the values of x, y, z, t for which the matrices $\begin{bmatrix} x+y & y-t \\ z+t & x+z \end{bmatrix}$ *and* $\begin{bmatrix} y-z & x-z \\ 2+t & 3+y \end{bmatrix}$ may be equal.

ii. find the inverse of $\begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 2 & 2 & -1 \end{bmatrix}$

f. i. Let the Linear transformations T_1 and T_2 from $R^2 \rightarrow R^2$ defined by T1(x,y)=(y,-x) and $T_2(x,y)=(y,0)$ Find the formula to define the mappings T_1T_2 and $T_1T_2 - T_2T_1$

ii. Show that the set of all polynomial in X over a field F of degree $\leq$ n is a subspace of the vector space of all polynomials over F.