BCA 1st Semester (Honours) Examination-2022-23

BACHELOR OF COMPUTER APPLICATION

(NEW SYLLABUS)

Course ID : 13314 **Course Code : GE-01**

Course Title : Mathematics-1

Time : 3 Hours *Full Marks :* 80

The figures in the right hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer ***all*** questions : 1×10=10

(a) If z = - 1 + i, then argument of z is

(i) $\frac{3\pi}{4}$

(ii) $-\frac{3\pi}{4}$

(iii) $\frac{3\pi}{2}$

(iv) $-\frac{3\pi}{2}$

(v) None of these

(b) Value of $\sqrt[3]{i} + \sqrt[3]{-i}$ is

(i) $\sqrt{3}$

(ii) $-\sqrt{3}$

(iii) $\frac{\sqrt{3}}{2}$

(iv) $-\frac{\sqrt{3}}{2}$

(v) None of these

(c) If a be a multiple root of 3 of the equation $x^4 + bx^2 + cx + d = 0$, $(d \neq 0)$, then a is

(i) $-\frac{7d}{3c}$

(ii) $-\frac{6d}{5c}$

(iii) $-\frac{8d}{3c}$

(iv) $\frac{8d}{3c}$

(v) None of these

(d) (i) Find the matrix A, if its adjoint is $\begin{bmatrix} -2 & 3 & 1 \\ 6 & -8 & -2 \\ -4 & 7 & 1 \end{bmatrix}$ and its determinant value is 2.

(ii) If the value of a tetrahedron be 2 units and three of its vertices be A(1,1,0), B(l,0,1), C(2, -1,1), then find the locus of the fourth vertex. 5+5

(e) (i) Find the unit vector perpendicular to both $a = 2\hat{i} + 3\hat{j} - \hat{k}$, and $b = 3\hat{i} - \hat{j} + 2\hat{k}$. Find also the angle between them. 3+2

(ii) If f : A $\rightarrow$ B and g: B $\rightarrow$ C be two mappings such that go$f$: A $\rightarrow$ C is surjective, then is it necessary f is surjective? Justify your answer.

(f) Identify the conic T given by F = 0, where F = $9x^2 - 24xy + 16y^2 - 18x - 10ly + 19$ and determines its vertices and axes. 2+4+4 find the matrix of T relative to the ordered bases

$((1,1,0), (1,0,1), (0,1,1))$ of $\mathbb{R}^3$ and $((1,1), (0,1))$ of $\mathbb{R}^2$.

5+2+3

(b) (i) Use De Moivre's theorem to prove that,

$\cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos\theta$

(ii) Find the product of all the values of $(1+i)^{\frac{4}{5}}$

(iii) Solve $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$

3+4+3

(c) (i) If α,β,γ are the roots of $px^3 + 3qx^2 + 3rx + s = 0$,

find $\sum \alpha^3$. 3+4+3

(ii) Show that the determinant $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$ is a

perfect square. 5+5

(d) The nature of the roots of the equation

$3x^4 + 12x^2 + 5x - 4 = 0$ is

(i) 2 real roots and 2 non-real roots

(ii) 1 real root and 3 non-real roots

(iii) 3 real roots and 1 non-real root

(iv) All non-real roots

(v) None of these

(e) If $l_n = \left[0, \frac{1}{n}\right]$, $n, \in \mathbb{N}$, then $\bigcap_{n \notin \mathbb{N}} l_n$ equal to

(i) $\{0\}$

(ii) $\varnothing$

(iii) (i) and (ii) both

(iv) 0

(v) None of these

(f) If $l_n = \left(0, \frac{1}{n}\right)$, $n, \in \mathbb{N}$, then $\bigcap_{n\in\mathbb{N}} l_n$ equal to

(i) (0)

(ii) $\varnothing$

(iii) (i) and (ii) both

(iv) 0

(v) None of these

(g) If $f = \{(x, y) \notin \mathbb{R}\times\mathbb{R} : y = \frac{1}{x}\}$, then

(i) f is a mapping from $\mathbb{R}$ to $\mathbb{R}$

(ii) f is a mapping from $\mathbb{N}$ to $\mathbb{R}$

(iii) f is not a mapping from $\mathbb{R}$ to $\mathbb{R}$

(iv) f is not a mapping from $\mathbb{N}$ to $\mathbb{R}$

(v) None of these

(e) Find $\dim(S \cap T)$, where S and T are subspaces of the vector space $\mathbb{R}^4$ given by

$$S = \left\{(x, y, z, w) \in \mathbb{R}^4 : 2x + y + 3z + w = 0\right\}$$

$$T = \left\{(x, y, z, w) \in \mathbb{R}^4 : x + 2y + z + 3w = 0\right\}$$

(f) Reduce the equation $16x^2 - 24xy + 9y^2 - 104x - 172y + 44 = 0$ to its canonical form and determine the nature of the conic. Find the equation of its axes. 2+3

Group – D

4. Answer **any *three*** questions : 10×3=30

(a) (i) If V is a vector space of dimension n over a field F, then prove that any linear independent set of n vectors of V is a basis of V.

(ii) The matrix of a linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^2$ relative to the order bases ((0,1,1), (1,0,1), (1,1,0)) of $\mathbb{R}^3$ and ((1,0), (1,1)) of $\mathbb{R}^2$ is $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \end{pmatrix}$. Find T. Also

Group – C

3. Answer **any *four*** questions : 5×4=20

(a) Find the vector area of the triangle, the position vectors of whose vertices are

$\left(\hat{i}+\hat{j}+2\hat{k}\right),\left(2\hat{i}+2\hat{j}+3\hat{k}\right)$ *and* $(3\hat{i}-\hat{j}-\hat{k})$.

(b) Show that

(i) $[\alpha+\beta,\ \beta+\gamma,\ \gamma+\alpha]=2[\alpha\beta\gamma]$

(ii) $[abc]^2=\begin{vmatrix} a.a & a.b & a.c \\ a.b & b.b & b.c \\ a.c & b.c & c.c \end{vmatrix}$ 3+2

(c) If H and K be finite subgroups of a group G such that HK is a group of G, then prove that

$$O(HK)=\frac{O(H).O(K)}{O(H\cap K)}$$

(d) Examine if the ring of matrices $\left\{\begin{pmatrix} a & b \\ 2b & a \end{pmatrix}: a,b\in\mathbb{R}\right\}$ is a field.

(h) If G = {–1,0,1}, then (G,+)

(i) Is a group

(ii) Is not a group

(iii) Is a semi-group

(iv) Is a monoid

(v) None of these

(i) If $\vec{a},\vec{b},\vec{c}$ *are three vectors, then value of* $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ is

(i) $\left(\vec{a}.\vec{c}\right).\vec{b}-\left(\vec{a}.\vec{b}\right).\vec{c}$

(ii) $\left(\vec{a}.\vec{b}\right).\vec{c}-\left(\vec{a}.\vec{c}\right).\vec{b}$

(iii) $\left(\vec{b}.\vec{c}\right).\vec{a}-\left(\vec{c}.\vec{a}\right).\vec{b}$

(iv) $\left(\vec{a}.\vec{c}\right).\vec{b}-\left(\vec{b}.\vec{c}\right).\vec{a}$

(v) None of these

(j) ($\mathbb{Z}$, +,.) is

(i) A field

(ii) Not a field

(iii) A commutative skew field

(iv) A non-commutative skew field

(v) None of these

Group – B

2. Answer **any *ten*** question : 2×10=20

(a) Find by Descarte's rule of sign, the nature of the roots of the equation $x^n = 1$, n being a natural number.

(b) Solve the equation $x^3 - 7x^2 + 19x - 13 = 0$, given that one of its roots is 3 + 2i.

(c) Find a complex number z for which $e^z = -i$.

(d) Solve $z^5 = 1$, where z is a complex number.

(e) Write the subset of the set {x, y, z}.

(f) State with reason whether the mapping $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x|x|, \forall x \in \mathbb{R}$ is injective, or not.

(g) Does the set {–2, –1,0,1,2} forms a group under usual addition? Justify your answer.

(h) In a group (G, o), a is an element of order 30. Find the order of a^{18}.

(i) Express (5,2,1) as a linear combination of (1,4,0), (2,2,1) and (3,0,1).

(j) Determine k, so that the vectors (1,3,1), (2,k,0) and (0,1,4) are linearly dependent.

(k) Solve the equation $2x^3 - x^2 - 18x + 9 = 0$, if two of the roots are equal in magnitude but opposite in sign.

(l) Show that the vectors corresponding to the positions of the point (3, –2, 1) and (2, 3, 0) are at right angles.

(m) Find the angle between planes

$$\vec{r}.\left(2\hat{i} + 3\hat{j} + \hat{k}\right) = 7 \text{ and } \vec{r}.\left(3\hat{i} - 2\hat{j} + 5\hat{k}\right) = 5.$$

(n) Transfer the equation

$3(12x - 5y + 39) + 2(5x + 12y - 26)^2 = 169.$

(o) Find the equation of the conic passing through the point of interesection of the straight lines $x - 3y - 4 = 0$ and $x + y = 0$ and the intersection of the conics $x^2 - 3xy + y^2 - 6x - 4y + 5 = 0$ and $3x^2 + 7xy - 3y^2 - 14x - 2y + 23 = 0$.