B.Sc. 1st Semester (Honours) Examination-2022-23

PHYSICS

Course ID : 12411 Course Code : SH/PHS/101/C-1

Course Title : Mathematical Physics-I (New)

Time : 1 Hour 15 Minutes *Full Marks :* 25

The figures in the right hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Unit–I

1. Answer any *five* questions : 1×5 =5

(a) $\vec{F}$ is a constant vector and $\vec{r}$ is the position vector then $\nabla\left(\vec{F}\cdot\vec{r}\right)=?$

(b) A force given by $\vec{F}=3\hat{i}+2\hat{j}-4\hat{k}$ is applied at the point (1,–1,2). Find the moment of force $\vec{F}$ about the point (2,–3,1).

(c) $\varphi=2e^{2x+y-z}$, Find $\vec{\nabla}\varphi$.

(d) Write down expression of elementary volume in spherical polar coordinate.

(e) If $\vec{A}$ is conservative, Evaluate $\oint \vec{A}.\overrightarrow{dr}$.

(f) State the divergence theorem.

(g) Express the point P(1, –4, –3) in cylindrical coordinates.

(h) Find $\Gamma(-3\cdot5)$.

Unit–II

2. Answer any *two* questions : 5×2 =10

(a) Prove that $\int_0^{\pi}\left[p_t(\cos\theta)\right]^2 \sin\theta d\theta = \frac{2}{2l+1}$.

(b) Let $f(x, t)$ be the solution of heat equation $\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial x^2}$ in one dimension. The initial condition at $t = 0$ is $f(x,0) = e^{-x^2}$ for $-\infty < x < \infty$. Then for all $t > 0$ find $f(x.t)$.

$\left[Given \int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}\right]$.

(c) Solve the differential equation

$$\frac{d^2y}{dx^2} - y = x \sin x + (1+x^2)e^x$$

(d) $i = \begin{cases} I_0 \sin x \; for \; 0 \le x \le \pi \\ 0 \; for \; 0 \le x \le 2\pi \end{cases}$

Express i as a Fourier series.

Unit–III

3. Answer any *one* question : 10×1 =10

(a) (i) Show that

$$\vec{F} = \left(y^2 \cos x + z^3\right)\hat{i} + \left(2y \sin x - 4\right)\hat{j} + \left(3xz^2 + 2\right)\hat{k}$$

is a conservative force field.

(ii) Find the scalar potential for $\vec{F}$.

(iii) Find the work done in moving an object in this field from (0,1,–1) to $\left(\frac{\pi}{2}, -1, 2\right)$.

(iv) Evaluate $\iint \vec{A}.\hat{n}dS$, where $\vec{A} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. 2+2+2+4

(b) (i) Prove that $\int_0^{\pi/2} \sqrt{Cot\theta}\; d\theta = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$

(ii) Using method of separation of variables find the solution of $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$; with the boundary condition U $(0, y) = 8e^{-3y}$. 5+5