(iii) Prove that the length of the focal chord of the conic $\frac{l}{r}=1-e\cos\theta$ which is inclined to the axis at an angle α is $\frac{2l}{1-e^2\cos^2\alpha}$.

(b) When is the differential equation $Mdx + Ndy = 0$ exact? If $Mx + Ny \neq 0$ then show that $\frac{1}{Mx+Ny}$ is an Integrating factor of the equation $Mdx + Ndy = 0$. Hence solve the equation $x^2ydx - (x^3 + y^3)dy = 0$.

B.Sc. 1st Semester (Programme) Examination-2022-23

MATHEMATICS

Course ID : 12118 Course Code : SP/MTH/101/C-1A

Course Title : Calculus and Geometry (New)

Time : 2 Hours *Full Marks :* 40

The figures in the right hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

Unit–I

1. Answer **any *five*** questions : 2×5=10

(a) If $P_n=\frac{d^n}{dx^n}\left(x^n\log x\right)$, then prove the recurrence relation $P_n=nP_{n-1}+(n-1)!$.

(b) Show that for the curve $\beta y^2 = (x+\alpha)^3$, the square of the subtangent varies as the subnormal.

(c) If $I_n=\int_0^a\left(a^2-x^2\right)^n dx$ and $n\in\mathbb{Z}$, prove that $I_n=\frac{2na^2}{2n+1}I_{n-1}$.

(d) Write down the general equation of second degree in two variables and mention, with justification, the number of independent arbitary constants involved in it.

(e) Find the area of the region bounded by $x = \pm 1, y = 0$ and $y = x^2$.

(f) Determine the nature of the surface represented by the equation $5x^2 + 6y^2 + 10z^2 + 30 = 0$.

(g) Find the cylindrical polar coordinates of a point whose rectangular Cartesian coordinates are (3,3,4).

(h) A ray of light travels along a line $y = 4$ and strikes the surface of curves $y^2 = 4(x + y)$. Find the equation of the line along which the reflected ray travels.

Unit–II

2. Answer **any *four*** questions : 5×4=20

(a) What do you mean by rectilinear asymptote of a curve? Determine the asymptotes of the curve $(x^2 - a^2)y^2 = x^2(x^2 - 4a^2)$. 1+4

(b) When an integral call improper? Define Gamma integral and state, with justification, whether it is improper or not. By using Beta and Gamma functions, evaluate the integral $\int_0^2 x^{\frac{5}{2}}(2-x)^{\frac{1}{2}}\,dx$. 1+(1+1)+2

(d) (i) Find the equation of the sphere through the four points (0,0,0), $(-a, b, c)$, $(a, -b, c)$, $(a, b, -c)$ and determine the radius.

(ii) Show that the straight line $\frac{l}{r} = A\cos\theta + B\sin\theta$ touches the conic $\frac{l}{r} = 1 + e\cos\theta$ if $(A-e)^2 + B^2 = 1$.

(e) If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, prove that

$$\left(x^2-1\right)y_{n+2} + (2n+1)xy_{n+1} + \left(n^2+1\right)y_n = 0.$$

(f) Find the asymptotes of the curve $x^3 - 2x^2y + xy^2 + x^2 - xy + 2 = 0$.

Unit–III

3. Answer **any *one*** question : 10×1=10

(a) (i) Show that the point in which the curve $y = c\sin(x/a)$ meets the axis of x is a point of inflexion. 3

(ii) Find the arc length of the curve $r = \theta^2$ between the points $\theta = 0$ and $\theta = \sqrt{5}$.

(g) Evaluate $\int_{0}^{\frac{\pi}{4}} \tan^{6} x dx$.

(h) Find the interval in which the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave downwards.

Unit–II

2. Answer **any *four*** questions : 5×4=20

(a) Write down the general form of 1st order linear ordinary differential equation. Solve the differential equation $\frac{dy}{dx} - y \tan x = e^x \sec x$.

(b) Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular line if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.

(c) If $I_n = \int_0^1 x^n \tan^{-1} x dx$, n being positive integer (n>2) then show that $(n+1) I_n + (n-2) I_{n-2} = \frac{\pi}{2} - \frac{1}{n}$.

(c) If PSQ and $PS'R$ be, respectively, the two chords of an ellipse through the foci S and S' ; then prove that $\left(\frac{SP}{SQ} + \frac{S'P}{S'R}\right)$ is independent of the position of P.

(d) Define intrinsic equation to a curve and hence find the intrinsic equation of the Cardioid $r = a(1 - \cos\theta)$, the arc being measured from the cusp where $\theta = \theta$. 1+4

(e) What are the necessary and sufficient conditions for the general equation of second degree in x and y so that it should represent a pair of straight lines? Show that the equation $2x^2 + 3xy + y^2 = 0$ represents a pair of straight lines and find the angle between them. 1+2+2

(f) A variable plane is parallel to the given plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ and meets the coordinates axes at the points A, B and C. Prove that the circle ABC lies on the cone $\left(\frac{b}{c} + \frac{c}{b}\right) yz + \left(\frac{c}{a} + \frac{a}{c}\right) zx + \left(\frac{a}{b} + \frac{b}{a}\right) xy = 0$. 2+3

Unit–III

3. Answer **any *one*** question : 10×1=10

(a) (i) State L' Hospital's theorem. Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$.

(ii) Show that the points of inflexion on the curve $y^2 = (x - a)^2(x - b)$ lie on the line $3x + a = 4b$.

(iii) Find the length of the loop of the curve

$x = t^2$, $y = t - \frac{t^3}{3}$. (1+3)+3+3

(b) (i) State and prove 'Reflection property of Ellipse'.

(ii) Define ruled surface and its generator. Show that perpendiculars from the origin to the generators of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ lie on the surface $\frac{a^2\left(b^2+c^2\right)^2}{x^2}+\frac{b^2\left(c^2+a^2\right)^2}{y^2}=\frac{c^2\left(a^2-b^2\right)^2}{z^2}$.

5+5

Course Title : Calculus and Geometry (Old)

Unit–I

1. Answer **any *five*** questions : 5×2=10

(a) Find the order and degree of the differential equation

$y = y'\left(x+\sqrt{yy'}\right)$ where $y' = \frac{dy}{dx}$.

(b) Find the area of the circle $r = 2a\sin\theta$.

(c) Show that $\lim_{x\to 0}\left(1+\frac{1}{x}\right)^x = 1$.

(d) Find the singular solution of the differential equation

$y = px + p, \left(p=\frac{dy}{dx}\right)$.

(e) Determine the nature of the conic

$x^2 + 4xy + y^2 - 2x + 2y + 6 = 0$.

(f) Find the points on the conic $\frac{15}{r}=1-4\cos\theta$ whose radius vector is 5.