**22-23/12114**

# B.Sc. 1st Semester (Honours) Examination-2022-23

## MATHEMATICS

**Course ID : 12114 Course Code : SH/MTH/103/GE-1**

**Course Title : Calculus, Geometry & Vector Analysis (New)**

*Time :* 2 Hours *Full Marks :* 40

*The figures in the right hand margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Notations and symbols have their usual meaning.*

### Unit–I

**1.** Answer **any *five*** questions : 2×5=10

(a) Find the co-ordinate of the points where the origin is to be shifted so that the equation $3x^2+8xy+3y^2-2x+2y-2 = 0$ can be reduced to one which is free from linear terms.

(b) Evaluate $\lim_{x \to 0}(1+\sin x)^{\cot x}$.

(c) Find the $n^{\text{th}}$ order derivative of $x^3 \log x$.

(d) Find $\int_0^{\frac{\pi}{2}} \sin^8 x \, dx$.

(e) Find the volume of the solid generated by the revolution of the area enclosed by the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the $x$ axis.

(f) show that $\left[\vec{\alpha}\times\vec{\beta}\ \vec{\beta}\times\vec{\gamma}\ \vec{\gamma}\times\vec{\alpha}\right] = \left[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}\right]^2$.

(g) For the curve $\vec{r} = 2a \cos t\, \hat{i} + 2a \sin t\, \hat{j} + bt^2\hat{k}$, show that $[\dot{r}\ \ddot{r}\ \dddot{r}] = 8a^2bt$.

(h) Find the radius and polar coordinates of the centre of the circle $r = 4\cos\theta + 3\sin\theta$.

## Unit–II

**2.** Answer any *four* questions : 4×5

(a) Give the definition of rectilinear asymptote and determine the asymptotes of the following algebraic curve $x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0$. 1+4

(b) If $\log y = \tan^{-1}x$ then show that

$(1 + x^2)y_{n+2}+(2nx+2x-1)y_{n+1} + n(n+1)y_n = 0$.





(f) Solve : $\frac{dy}{dx} + y\cos x = y^n \sin 2x$.

## Unit–III

**3.** Answer **any *one*** question : 10×1=10

(a) (i) Define generating line of the surface. Find the equation of the generators of the hyperboloid $x^2 - y^2 = 4z$, which passes through the point (7,5,6).

(ii) Find $a$ and $b$ in order that $\lim_{x\to 0}\frac{a\sin 2x - b\sin x}{x^3} = 1$.

(iii) If $f(x)=x^m$ then prove that nth ($n>m$) order derivative of $f(x)$ is zero. 5+3+2

(b) (i) Prove that the envelope of the family of circle passing through the origin and having centres situated on the hyperbola $x^2 - y^2 = c^2$ is $(x^2 + y^2)^2 = 4c^2 (x^2 - y^2)$.

(ii) Find the general solution and singular solution of the differential equation $y = px + \sqrt{1+p^2}$.

(iii) Solve the differential equation

$\frac{dy}{dx} + yf'(x) = f(x)f'(x)$. 4+4+2

(f) Solve $\frac{dy}{dx} = (x^2+1)(y^2+1)$.

(g) Find an integrating factor of $(x^2 + y^2 + 2x)dx + 2y\, dy = 0$.

(h) Find the polar equation of parabola whose latus rectum is 6.

**Unit–II**

**2.** Answer **any *four*** questions : 5×4=20

(a) Reduce the equation $x^2 + 4xy + y^2 - 2x + 2y + 6 = 0$ to its canonical forms and determine the nature of the conic.

(b) (i) If $y = \cos(10\cos^{-1}x)$ then show that $(1 - x^2)y_{12} - 21xy_{11} = 0$.

(ii) If $y = \sin^3 x$ then find $\frac{d^n y}{dx^n}$.

(c) Prove that the asymptotes of the curve $x^2y - y^3 - 2ay^2 + 5x - 7 = 0$ forms a triangle of area $a^2$.

(d) Find the equation of the sphere touching the three co-ordinate planes.

(e) Show that the equation of the tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$ parallel to the tangent at $\theta = \alpha$ is given by $l(e^2 + 2e\cos\alpha + 1) = r(e^2 - 1)[\cos(\theta - \alpha) + e\cos\theta]$.





(c) Reduce the equation $4x^2 - 4xy + y^2 + 2x - 26y + 9 = 0$ to its suitable form to determine the conic.

(d) A sphere of constant radius $r$ passes through the origin $O$ and cuts the axes in $A$, $B$, $C$. Prove that the locus of the foot of the perpendicular from $O$ to the plane $ABC$ is given by $(x^2 + y^2 + z^2)^2(x^{-2} + y^{-2} + z^{-2}) = 4r^2$.

(e) (i) Prove a necessary and sufficient conditions that a proper vector $\vec{u}$ has a constant length is that

$$\vec{u}\frac{d\vec{u}}{dt} = 0.$$

(ii) If $\vec{\alpha} = t\hat{i} + t^2\hat{j} + t^3\hat{k}$ then find the value of

$$\vec{a} \times \frac{d^2\vec{a}}{dt^2} \text{ at } t = 1.$$ 3+2

(f) (i) Evaluate $\int_2^3 \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2}\right)dt$ where $\vec{r} = t^3\hat{i} + 2t^2\hat{j} + 3t\hat{k}$.

(ii) If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be coplanar vectors, then show that

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}.$$ 3+2

## Unit–III

**3.** Answer any *one* question : 1×10

(a) (i) If $y = (x^2 - 1)^n$, then show that

$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n + 1)y_n = 0.$

(ii) Find the volume of the solid generated by revolving the cardioids $r = a(1-\cos\theta)$ about the initial line.

(iii) Find the point of inflexion of the curve $x = (\log y)^3$, if any. 4+3+3

(b) (i) State and prove the reflection property of parabola.

(ii) If $I_n = \int \sin^n\theta d\theta$ then show that $nI_n = \sin h^{n-1}\theta \cos h\theta - (n - 1)I_{n-2}$.

(ii) Prove that $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{b} \times \vec{a}$ intersect and find their intersection. 4+3+3

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## Course Title : Calculus, Geometry & Differential Equation (Old)

## Unit–I

**1.** Answer **any *five*** questions : 2×5=10

(a) Find the co-ordinate of the points where the origin is to be shifted so that the equation $2x^2 - 3xy + 4y^2 + 10x - 19y + 23 = 0$ can be reduced to one which is free from linear terms.

(b) If $I_n = \int_0^{\frac{\pi}{2}} x^n \sin xdx$ and $n>1$, show that

$I_n + n(n - 1)I_{n-2} = n(\pi /2)^{n-1}.$

(c) Prove that the curve $y = e^{2x}$, is always convex to $x$ axis at every point.

(d) If $\lim_{x\to c}\{f(x)/h(x)\}$ exists and is finite and if $\lim_{x\to c} h(x) = 0$ then prove that necessarily $\lim_{x\to c} f(x) = 0$.

(e) Show that $y = 3(x + 1)$ is an oblique asymptote of the curve $y = 3x + \frac{3x}{x-1}$.