(iii) Show that xy^2 is an integrating factor of $2ydx + 3xdy = 0$. 4+4+2

(b) (i) Find the perimeter of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$.

(ii) Find the reduction formula for $\int \frac{dx}{(x^2+a^2)^n}$ where n is a positive integer and hence find $\int \frac{dx}{(x^2+a^2)^3}$.

(iii) Solve : $\frac{dy}{dx} + xy = y^2 e^{x^2/2} \sin x$. 3+4+3

B.Sc. 1st Semester (Honours) Examination-2022-23

MATHEMATICS

Course ID : 12111 Course Code : SH/MTH/101/C-1

Course Title : Calculus, Geometry & Vector Analysis (New)

Time : 2 Hours *Full Marks* : 40

The figures in the right hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

Unit–I

1. Answer **any *five*** questions : 2×5=10

(a) Find $\lim_{x\to 0} \frac{x - \sin x}{\tan^2 x}$.

(b) Find the pedal equation of the curve $r^m = a^m \cos m\theta$, where m is a constant.

(c) If incident ray from the point (–1,2) parallel to the axis of the parabola $y^2 = 4x$ strikes the parabola, then find the equation of the reflection ray.

(d) Determine the length of one arch of the cycloid $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$

(Turn Over)

(e) Suppose $\vec{F}$ is a conservative field, then prove $\vec{F}$ that is irrotational.

(f) Prove that $\left(\vec{A}\times\vec{B}\right).\left(\vec{B}\times\vec{C}\right)\times\left(\vec{C}\times\vec{A}\right)=\left(\vec{A}.\vec{B}\times\vec{C}\right)^2$

(g) If $I_n=\int_0^a\left(a^2-x^2\right)^n dx$ and $n\in\mathbb{Z}$, prove that

$$I_n=\frac{2na^2}{2n+1}I_{n-1}.$$

(h) Find the vertical asymptotes of the curve

$$y=(a-x)\tan\left(\frac{\pi x}{2a}\right)$$

Unit–II

2. Answer **any *four*** questions : 5×4=20

(a) Define Curvature of a curve and show that the radius of curvature at (r,θ) of the polar curve $r^2=a^2\cos 2\theta$ is $\dfrac{a^2}{3r}$. 1+4

(b) Show that the volume of the spindle formed by revolution of a parabolic arc about the line joining vertex to one extremity of the latus rectum is $\dfrac{2\pi a^3\sqrt{5}}{75}$ where $4a$ is the length of latus rectum.

(d) (i) Solve : $\left(x^2+y^2+1\right)dx+x\left(x-2y\right)dy=0$

(ii) Solve : $\left(1+y^2\right)dx=\left(\tan^{-1}y-x\right)dy$ 3+2

(e) Find the co-ordinates of the centre and radius of the circle $x^2+y^2+z^2-2y-4z-11=0,\ x+2y+2z=15$.

(f) Reducing the equation $4x^2+4xy+y^2-4x-2y+a=0$ to its canonical form, determine the nature of the conic for different values of a.

Unit–III

3. Answer **any *one*** question : 10×1=10

(a) (i) Find the asymptotes of the equation

$y^3-5xy^2+8x^2y-4x^3-4y^2+12xy-8x^2+3y-3x+2=0.$

(ii) Evaluate : $\lim\limits_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{\frac{1}{x}}$.

(f) Obtain the centre and the radius of the sphere $3x^2 + 3y^2 + 3z^2 + 2x - 4y - 2z - 1 = 0$.

(g) Obtain the equation of the cone, obtain by revolving the straight line $x - 2y - 1 = 0$ in the plane $z = 0$ about the x-axis.

(h) Obtain the general solution of the differential equation $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} - 12y = 0$.

Unit–II

2. Answer **any *four*** questions : 4×5

(a) If $y = sin\,(m\,sin^{-1}x)$, show that $(1 - x^2)y_{n+2} = (2n + 1)xy_{n+1} + (n^2 - m^2)y_n$. Also Find $y_n(0)$.

(b) Prove that $\int_0^{\frac{\pi}{2}} \sin^n x\, dx = \frac{n-1}{n}\,\frac{n-3}{n-2}\cdots\cdots\frac{3}{4}\,\frac{1}{2}\,\frac{\pi}{2}$, when n is even.

(c) Show that the curve $(1 + x^2)y = 1 - x$ has three points of inflexion and that they lie on a straight line.

(c) What are the necessary and sufficient conditions for the general equation of second degree in x and y so that it should represent a pair of straight lines? Show that the equation $2x^2+3xy+y^2=0$ represents a pair of straight lines and find the angel between them. 1+2+2

(d) Find a reduction formula for $I_{m,n} = \int_0^{\pi/2} cos^m x sinx\, dx$, m,n being positive integers and hence deduce that

$$I_{m,n} = \frac{1}{2^{m+1}}\left[2 + \frac{2^2}{2} + \frac{2^3}{3} + \cdots + \frac{2^m}{m}\right]$$ 2+3

(e) (i) If $\vec{r} = a\cos t\,\hat{i} + a\sin\,\hat{j} + at\tan a\,\hat{k}$, then find $\left|\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right|$.

(ii) Evaluate $\int \vec{u}.\left(\vec{v} \times \frac{d^2\vec{v}}{dt^2}\right)dt$, where $\vec{u}$ is a constant vector. 2+3

(f) Determine the position and nature of the multiple point of the curve $x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$. Also find the tangents at the multiple point, if any. 3+2

Unit–III

3. Answer **any *one*** question : 10×1=10

(a) (i) State Leibnitz's theorem on successive differentiation. If $f(x)=x^n$, prove that

$$f(1)+f'(1)+\frac{f''(1)}{2!}+\frac{f'''(1)}{3!}+\cdots+\frac{f^n(1)}{n!}=2^n,\ n\in\mathbb{N}$$

(ii) Define node and cusp points of an algebraic curve. Show that the origin is a node, a cusp or an isolated point on the curve $2y^2 = ax^2 + bx^3$ according a >, = or < 0.

(iii) Find the length of the loop of the curve $x=t^2, y=t-\frac{t^3}{3}$. (1+2)+(2+2)+3

(b) (i) If the straight line $\frac{x}{1}=\frac{y}{z}=\frac{z}{3}$ represents one of a set of three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$, then find the equations of the other two.

(ii) A plane passing through a fixed point (a,b,c) cuts the axes in A,B,C. Show that the locus of the centre of the sphere $OABC$ is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$. 5+5

Course Title : Calculus, Geometry & Differential Equation (Old)

Unit–I

1. Answer **any *five*** questions : 2×5=10

(a) Using L' Hospital rule find $\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right)$.

(b) If m is a positive integer greater than n, then prove that $D^n(ax+b)^m=\frac{m!}{(m-n)!}a^n(ax+b)^{m-n}$, where $D\equiv\frac{d}{dx}$.

(c) Find the envelope of the straight line $y=mx+a\sqrt{1+m^2}$, where m is a parameter.

(d) Find a reduction formula for $I_m=\int x^m e^x dx$ and hence Evaluate I_3.

(e) Prove that the differential equation $Mdx + Ndy = 0$ possess and infinite number of integrating factor.